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FAST TRACK COMMUNICATION

Two interacting atoms in a cavity: exact solutions, entanglement and decoherence

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Abstract
We address the problem of two interacting atoms of different species inside a cavity and find the explicit solutions of the corresponding eigenvalues and eigenfunctions using a new variant. This model encompasses various commonly used models. By way of example we obtain closed expressions for concurrence and purity as a function of time for the case where the cavity is prepared in a number state. We discuss the behaviour of these quantities and their relative behaviour in the concurrence–purity plane.

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(Some figures in this article are in colour only in the electronic version)

The system of two two-level atoms (TLA) inside a cavity has attracted considerable attention, both because it has become experimentally feasible and because it is the paradigm to study the evolution of entanglement under decoherence. This combination is remarkable because entanglement is a central resource and decoherence the major impediment for quantum information processing [1]. The relation between concurrence and purity of the central system yields the simplest access to the problem.

Different models of two identical TLA as a central system coupled to a cavity mode in resonance with the atomic transition as the environment have been studied [2–5]. In this communication, we show that one can define a wider class of such systems that remains solvable in closed form and includes the above mentioned cases. Specifically we consider atoms with different coupling to the cavity mode, different detuning and include dipole–dipole as well as Ising interactions between the atoms.

We show that the total number of excitations is a conserved quantity. Using the basis in which the corresponding operator is diagonal, the Hamiltonian will be transformed to the block diagonal form, with maximally 4×4 blocks. Interestingly we could use a special case of this solution to construct an exactly solvable relativistic model [10] with three degrees of freedom, namely a Dirac oscillator [11] coupled to an isospin field.

Note that models with different interacting TLA and a single excitation on a continuum of modes have been solved [6–8] using the *pseudomode* approach [9] which in those cases results in a single mode with losses. While being quite similar, the loss term violates the conservation law we use and thus these are not particular cases of our model.

In order to focus on a particular new aspect, namely the interplay of Ising and dipole–dipole interactions, we shall choose an example where other features of our model are simplified. Thus, we shall apply the closed solution to study the time evolution of concurrence and purity of two interacting TLA with equal coupling and zero detuning but arbitrary dipole–dipole and Ising interactions. The interaction-free case basically provides the borders of the evolution if we look at the interacting problem in a concurrence–purity (CP) diagram, a third boundary being provided by the relative strength of the two interactions.

Particular cases of the general model, for which solutions are available, should be experimentally feasible in cavity QED [12, 13]. While dipole–dipole interactions commonly appear in QED, an Ising interaction might be simulated as proposed in [14–16]. Whether a particular case, such as the one we discuss, will actually be measured depends on specific difficulties in forming the initial state, as well as the amount of interest such a case may bring. Some such cases, including initial coherent states, will be studied in a forthcoming paper [17].

Consider the Hamiltonian for two TLA coupled to a cavity mode and set $\hbar = 1$; we use the rotating wave approximation and work in the interaction picture so we end up with

$$H = \sum_{j=1}^{2} \left\{\delta_j \sigma_z^{(j)} + g_j\left(a\,\sigma_+^{(j)} + a^\dagger \sigma_-^{(j)}\right)\right\} + 2\kappa\left(\sigma_-^{(1)}\sigma_+^{(2)} + \sigma_+^{(1)}\sigma_-^{(2)}\right) + J\sigma_z^{(1)}\sigma_z^{(2)}, \tag{1}$$

where δ_j is the detuning of the corresponding atomic transition frequency from the frequency of the cavity mode which does not appear due to our choice of the interaction picture.

g_j is the coupling to the mode, and κ and J are the strengths of the dipole–dipole and Ising interactions, respectively. We use the standard definitions of creation and annihilation operators for the harmonic oscillator $(a, a^\dagger)$ and for the raising and lowering operators $\sigma_\pm = (\sigma_x \pm \mathrm{i}\sigma_y)/2$, with the Pauli matrices $(\sigma_x, \sigma_y, \sigma_z)$.

The operator $I = a^\dagger a + 1/2\left(\sigma_z^{(1)} + \sigma_z^{(2)}\right)$ provides an additional constant of motion and it can be interpreted as the number of excitations in the system. Clearly, $[H, I] = 0$ and in general this is the only commuting observable of this problem. Therefore, we choose the following basis for which I is diagonal:

$$\begin{aligned} \left|\phi_1^{(n)}\right\rangle &= |n+1\rangle|{-}{-}\rangle & \left|\phi_2^{(n)}\right\rangle &= |n\rangle|{-}{+}\rangle \\ \left|\phi_3^{(n)}\right\rangle &= |n\rangle|{+}{-}\rangle & \left|\phi_4^{(n)}\right\rangle &= |n-1\rangle|{+}{+}\rangle. \end{aligned} \tag{2}$$

Here $|n\rangle$ describes a state of n photons in the cavity, and $|-\rangle$ and $|+\rangle$ describe the ground and excited states of a TLA respectively. For any given n they satisfy the relation $I\left|\phi_j^{(n)}\right\rangle = n\left|\phi_j^{(n)}\right\rangle$. In this basis H is a block-diagonal matrix and each block $H^{(n)}$ is a 4×4 matrix with elements $\left\langle\phi_j^{(n)}\right|H\left|\phi_k^{(n)}\right\rangle \equiv H_{jk}^{(n)}$. Explicitly, one has

$$H^{(n)} = \begin{pmatrix} J - \delta_1 - \delta_2 & g_2\sqrt{n+1} & g_1\sqrt{n+1} & 0 \\ g_2\sqrt{n+1} & \delta_2 - \delta_1 - J & 2\kappa & g_1\sqrt{n} \\ g_1\sqrt{n+1} & 2\kappa & \delta_1 - \delta_2 - J & g_2\sqrt{n} \\ 0 & g_1\sqrt{n} & g_2\sqrt{n} & J + \delta_1 + \delta_2 \end{pmatrix}. \tag{3}$$

For $n = 0$, the basis is reduced to the three states $|1\rangle|{-}{-}\rangle$, $|0\rangle|{-}{+}\rangle$ and $|0\rangle|{+}{-}\rangle$. For $n = -1$ it is reduced to one single $|0\rangle|{-}{-}\rangle$. This single state is stationary and represents the situation where both atoms are in the ground state and there are no photons in the cavity.

Solving the resulting eigenvalue problem implies diagonalizing each block of the Hamiltonian. In general the characteristic polynomial for the eigenvalues leads to a depressed quartic equation with eigenvalues:

$$E_j^{(n)} = \begin{cases} -\frac{\sqrt{R+U}}{2} + \frac{(-1)^j}{2}\sqrt{2R - U + \frac{Q}{\sqrt{R+U}}}, & j = 1, 2 \\ \frac{\sqrt{R+U}}{2} + \frac{(-1)^j}{2}\sqrt{2R - U - \frac{Q}{\sqrt{R+U}}}, & j = 3, 4, \end{cases} \tag{4}$$

where we used the following definitions:

$$\begin{aligned} P &= \left(\delta_1^2 - \delta_2^2 + (n+1)\left(g_1^2 - g_2^2\right)\right)\left(\delta_1^2 - \delta_2^2 + n\left(g_1^2 - g_2^2\right)\right) \\ &\quad + J^2\left((2n+1)\left(g_1^2 + g_2^2\right) - 2\left(\delta_1^2 + \delta_2^2\right) + J^2 - 4\kappa^2\right) \\ &\quad + 2J\left(g_1^2\delta_1 + g_2^2\delta_2 + 2(2n+1)\kappa g_1 g_2\right) \\ &\quad + 4\kappa\,(\delta_1 + \delta_2)\,(g_1 g_2 + \kappa\,(\delta_1 + \delta_2)) \\ Q &= 4\left(g_1^2\delta_2 + g_2^2\delta_1 + 4J(\kappa^2 - \delta_2\delta_1) - 2(2n+1)\kappa g_1 g_2\right) \\ R &= 2/3\left((2n+1)\left(g_1^2 + g_2^2\right) + 2\left(\delta_1^2 + \delta_2^2 + J^2\right) + 4\kappa^2\right) \\ S &= 2PR + \frac{Q^2 - R^3}{8}, \qquad T = \frac{4P}{3} + \frac{R^2}{4} \\ U &= (S + \sqrt{S^2 - T^3})^{1/3} + (S - \sqrt{S^2 - T^3})^{1/3}. \end{aligned} \tag{5}$$

The eigenvectors before normalization read

$$\begin{aligned} v_{1,j}^{(n)} &= \left(E_j^{(n)} - \delta_1 - \delta_2 - J\right)\left(\left(E_j^{(n)} + J\right)^2 - n\left(g_1^2 + g_2^2\right) - (\delta_1 - \delta_2)^2 - 4\kappa^2\right) \\ &\quad - 2n\left((\delta_1 + J)g_2^2 + (\delta_2 + J)g_1^2 + 2\kappa g_1 g_2\right) \\ v_{2,j}^{(n)} &= \sqrt{n+1}\left(2\kappa g_1\left(E_j^{(n)} - \delta_1 - \delta_2 - J\right) + g_2\left(\left(E_j^{(n)} - \delta_1\right)^2 + n\left(g_1^2 - g_2^2\right) - (\delta_2 + J)^2\right)\right) \\ v_{3,j}^{(n)} &= \sqrt{n+1}\left(2\kappa g_2\left(E_j^{(n)} - \delta_1 - \delta_2 - J\right) + g_1\left(\left(E_j^{(n)} - \delta_2\right)^2 + n\left(g_2^2 - g_1^2\right) - (\delta_1 + J)^2\right)\right) \\ v_{4,j}^{(n)} &= \sqrt{n(n+1)}\left(2g_1 g_2\left(E_j^{(n)} + J\right) + 2\kappa\left(g_1^2 + g_2^2\right)\right) \end{aligned} \tag{6}$$

and the orthogonal transformation which diagonalizes the Hamiltonian is given by $V_{j,k}^{(n)} = v_{j,k}^{(n)} / \left(\sum_l v_{l,k}^{(n)\,2}\right)^{1/2}$.

By way of example we now treat a special case where we calculate the entanglement and purity of the pair of atoms considering the cavity mode as the environment. For this purpose it is convenient to start with product states of cavity and central system functions. We restrict ourselves to a definite value of the observable I and choose a number state for the cavity, i.e.

$$|\Psi_0\rangle = |n\rangle\,(\cos(\alpha)|{-}{+}\rangle + \sin(\alpha)|{+}{-}\rangle)\,. \tag{7}$$

In the same subspace of fixed eigenvalue of I, one could also use the state $|n+1\rangle|{-}{-}\rangle$ or $|n-1\rangle|{+}{+}\rangle$ as the initial product states. This type of initial state guarantees that the evolution stays confined in a four-dimensional subspace. The time evolution of the state vector under Hamiltonian (1) can be written as

$$|\Psi(t)\rangle = \sum_{l=1}^{4} B_l^{(n)}(t)\left|\phi_l^{(n)}\right\rangle \tag{8}$$

with the following coefficients:

$$B_k^{(n)} = \sum_{k=1}^{4} V_{l,j}^{(n)} \mathrm{e}^{-\mathrm{i}E_j^{(n)}t}\left(V_{2,j}^{(n)}\cos(\alpha) + V_{3,j}^{(n)}\sin(\alpha)\right). \tag{9}$$

For readability, we shall omit the time dependence in the coefficients, $B_k^{(n)} = B_k^{(n)}(t)$.

Starting from the density matrix of the whole system $\varrho(t) = |\Psi(t)\rangle\langle\Psi(t)|$, we take a partial trace over the cavity degree of freedom to compute the reduced density matrix of the two TLA, given by

$$\rho = \begin{pmatrix} \left|B_1^{(n)}\right|^2 & 0 & 0 & 0 \\ 0 & \left|B_2^{(n)}\right|^2 & \left(B_3^{(n)}\right)^* B_2^{(n)} & 0 \\ 0 & \left(B_2^{(n)}\right)^* B_3^{(n)} & \left|B_3^{(n)}\right|^2 & 0 \\ 0 & 0 & 0 & \left|B_4^{(n)}\right|^2 \end{pmatrix}. \tag{10}$$

The purity $P = \operatorname{Tr}\rho^2$ measures the entanglement between the central system and the environment, i.e. the decoherence of the two TLA and we find

$$P = \left|B_1^{(n)}\right|^4 + \left|B_4^{(n)}\right|^4 + \left(1 - \left|B_1^{(n)}\right|^2 - \left|B_4^{(n)}\right|^2\right)^2. \tag{11}$$

The concurrence [18] is used to measure the entanglement between the atoms. It is defined as $C(\rho) = \operatorname{Max}\{0, \lambda_1 - \lambda_2 - \lambda_3 - \lambda_4\}$, where λ_j are the eigenvalues of $\left(\rho\sigma_y^{(1)}\sigma_y^{(2)}\rho^*\sigma_y^{(1)}\sigma_y^{(2)}\right)^{1/2}$ in a non-increasing order. In our case the concurrence is given by

$$C(\rho) = \operatorname{Max}\left\{0, 2\left|B_2^{(n)}\right|\left|B_3^{(n)}\right| - 2\left|B_1^{(n)}\right|\left|B_4^{(n)}\right|\right\}. \tag{12}$$

Some interesting features can already be inferred by inspecting (11) and (12). For $n = 0$ we have $B_4^{(0)} = 0$ and the purity has a minimum value of $1/2$. As for the concurrence one can note the absence of entanglement sudden death [19, 20] in that particular case.

Now we specialize in the symmetric case with equal couplings to the cavity, zero detunings, but allow both types of interactions between the atoms. With these restrictions we are able to find explicit solutions in the time domain. Using the definitions

$$\begin{aligned} \omega_n &= \sqrt{4n + 2 + (\kappa - J)^2} \\ \beta_n &= \sqrt{\frac{4n^2 + 4n}{4n^2 + 4n + 1}}, \qquad \gamma_n = \frac{6n^2 + 6n + 2}{4n^2 + 4n + 1}, \end{aligned} \tag{13}$$

where ω_n is a frequency closely related to the eigenvalues of Hamiltonian (1), and the time-dependent functions

$$\begin{aligned} F(t) &= \frac{2n + 1}{\omega_n^2}(1 + \sin(2\alpha))\sin^2(\omega_n t) \\ G(t) &= \frac{(\kappa - J)\cos((J + 3\kappa)t)\sin(\omega_n t)}{\omega_n} + \sin((J + 3\kappa)t)\cos(\omega_n t), \end{aligned} \tag{14}$$

we find the following solutions for the purity and the concurrence as functions of time:

$$\begin{aligned} C(t) &= \operatorname{Max}\left\{0, \sqrt{(\sin(2\alpha) - F(t))^2 + \cos^2(2\alpha)G^2(t)} - \beta_n F(t)\right\} \\ P(t) &= 1 - 2F(t) + \gamma_n F^2(t). \end{aligned} \tag{15}$$

In figure 1 we present these solutions for the situation where there are no initial photons in the cavity, namely $n = 0$. We present three cases: in red and blue for non-interacting atoms with initial states defined by $\alpha = \pi/4$ and $\alpha = \pi/20$ respectively and an interacting case with $\kappa = 1.5$, $J = 0$ and $\alpha = \pi/20$ shown in black. Both quantities, concurrence and purity, display oscillatory behaviour, with one frequency in the non-interacting case and two frequencies in the interacting case as can be verified in equations (14) and (15). One can also note that with interaction (black curve) the concurrence increases while the minimum value of purity is greater in contrast to the corresponding non-interacting case (blue-dashed curve).

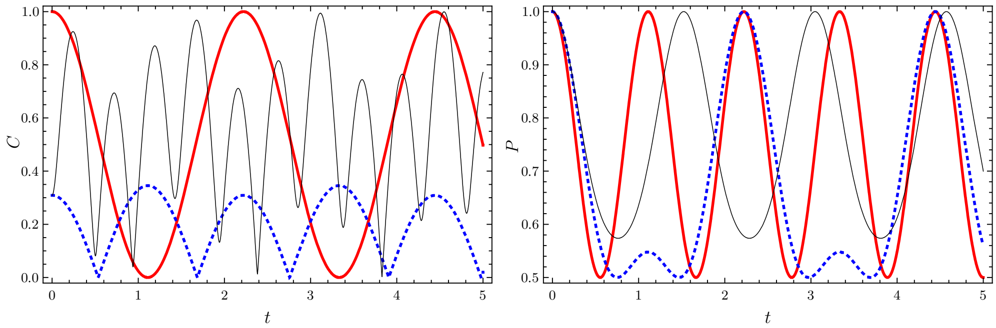

Figure 1. Concurrence and purity as functions of time for $n = 0$ and an initially empty cavity. The red curve corresponds two non-interacting atoms with an initial state determined by $\alpha = \pi/4$ (see equation (7)), i.e. a maximally entangled pure state. The blue-dashed curve shows the behaviour for non-interacting atoms with an initial pure, but not maximally entangled state with $\alpha = \pi/20$. In black, the curve for two interacting atoms with the same initial state as in the blue-dashed curve and $\kappa = 1.5$ and $J = 0$.

Similar behaviour in the time domain has already been studied in other references such as [2–5].

The graphs in the time domain look pretty standard and this does not change if both interactions are present. It is therefore convenient to visualize the joint dynamics in a concurrence versus purity plane, the CP-plane. Figure 2(*a*) shows the corresponding plane for the curves in figure 1 with the same colour code, but now the black curve is parametrized up to $t = 20$. In this plane we have plotted, to guide the eye, a grey zone corresponding to the concurrence and purity combinations that cannot be obtained in physical states and its lower frontier corresponds to the *maximally entangled mixed states* (MEMS), which for a given value of the purity maximize the concurrence [21]. The grey dashed line is defined by the Werner states $\rho_W = \xi \frac{I}{4} + (1-\xi)|\text{Bell}\rangle\langle\text{Bell}|$, $0 \leqslant \xi \leqslant 1$ [21, 22].

One can note as well that the dynamic of the interacting (black) case is enclosed by the non-interacting curves, the lower bound given by the blue curve with the same initial state as the black one, while the upper bound given by an initial state given by $\alpha = \pi/4$. Perhaps the most important feature here, that one cannot easily visualize in the time domain, is that for an initial bell state with $\alpha = \pi/4$ and no interaction between the atoms, the curve (red) follows precisely, as we shall prove below, the one that determines the mentioned MEMS. For this we need to first obtain the analytic solutions in the CP-plane.

We take the explicit solutions in time in equations (15), with $\kappa = J = 0$, and invert them to find an explicit relation of the concurrence in terms of the purity. In this non-interacting case, concurrence is represented by up to two different curves in the CP-plane

$$C_{\pm}^{(n)}(P;\alpha) = \mathrm{Max}\left\{0, \left|\sin(2\alpha) - f_{\pm}^{(n)}(P)\right| - \beta_n\, f_{\pm}^{(n)}(P)\right\} \tag{16}$$

with γ_n and β_n as given in equation (14) and with

$$f_{\pm}^{(n)}(P) = \frac{1 \pm \sqrt{1+\gamma_n(P-1)}}{\gamma_n}. \tag{17}$$

We find two separate cases.

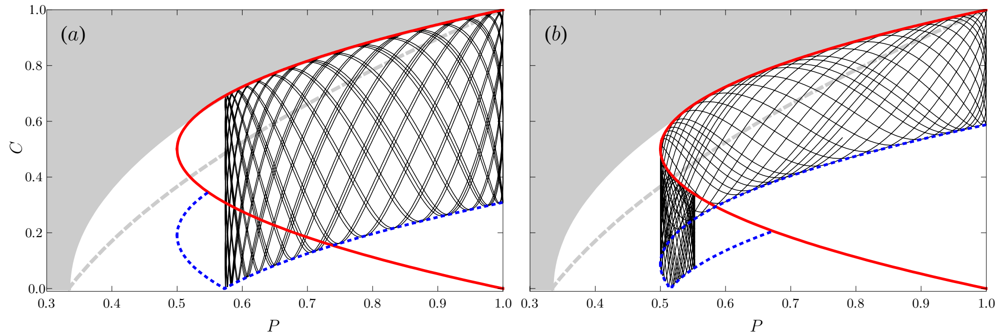

Figure 2. CP-plane for $n = 0$ and an initially empty cavity. The red curve shows the behaviour in the CP-plane for two non-interacting atoms with an initial state determined by $\alpha = \pi/4$ (see equation (7)), i.e. a maximally entangled pure state. The blue-dashed curve shows the behaviour for non-interacting atoms with an initial pure, but not maximally entangled state (*a*) $\alpha = \pi/20$ and (*b*) $\alpha = \pi/10$. In black, the curve for two interacting atoms with the same initial state as in the blue-dashed curve and parametrized by time up to $t = 20$. (*a*) $\kappa = 1.5$ and $J = 0$. (*b*) $\kappa = 1.5$ and $J = 0.87$. The grey area indicates CP combinations that cannot be obtained in physical states and its lower frontier corresponds to the *maximally entangled mixed states*. The grey dashed line represents the Werner states.

(i) For $\frac{1}{2}\arcsin\left(\frac{n^2+n}{3n^2+3n+1}\right) < \alpha < \pi/4$, the concurrence in the CP-plane is determined by the two curves:

$$\begin{aligned} &C_+^{(n)}(P;\alpha), \qquad 1-\frac{1}{\gamma_n} \leqslant P \leqslant \frac{\gamma_n\,(1+\sin(2\alpha))^2}{4} - \sin(2\alpha) \\ &C_-^{(n)}(P;\alpha), \qquad 1-\frac{1}{\gamma_n} \leqslant P \leqslant 1. \end{aligned} \tag{18}$$

(ii) Otherwise, the concurrence is determined only by the curve

$$C_-^{(n)}(P;\alpha), \qquad \frac{\gamma_n\,(1+\sin(2\alpha))^2}{4} - \sin(2\alpha) \leqslant P \leqslant 1. \tag{19}$$

In figure 2 we show these solutions in the CP-plane for $n = 0$ and different values of α. The red curve shows the case when the starting state is the symmetric Bell state, $\alpha = \pi/4$. This solution has the explicit form

$$C_\pm^{(0)}(P;\pi/4) = \tfrac{1}{2}(1 \mp \sqrt{2P-1}) \tag{20}$$

and it can be seen that in a certain region it coincides with the curve for the MEMS. In fact $C_-^{(0)}(P;\pi/4)$ coincides precisely with the curve of the MEMS for $5/9 \leqslant P \leqslant 1$. The dashed blue curve represents the situation with an initial state determined by a pure but not fully entangled state.

The behaviour for $n > 0$ is qualitatively the same and when one takes as initial state the symmetric Bell state, $\alpha = \pi/4$, the curves converges ($n \to \infty$) to

$$\begin{aligned} &C_-^{(\infty)}(P;\pi/4) = \mathrm{Max}\left\{0, \tfrac{1}{3}(\sqrt{24P-8}-1)\right\}, \qquad \tfrac{1}{3} \leqslant P \leqslant 1 \\ &C_+^{(\infty)}(P;\pi/4) = 0, \qquad \tfrac{1}{3} \leqslant P \leqslant \tfrac{1}{2}. \end{aligned} \tag{21}$$

In fact, in figure 3 we took $n = 5$ and the red curve is a very good approximation to $C_\pm^{(\infty)}$. We note however that in the limit $n \to \infty$ this curve, which is actually an upper bound, lies below the Werner curve. For finite n the curves $C_-^{(n)}(P;\pi/4)$ intersect the Werner curve in an

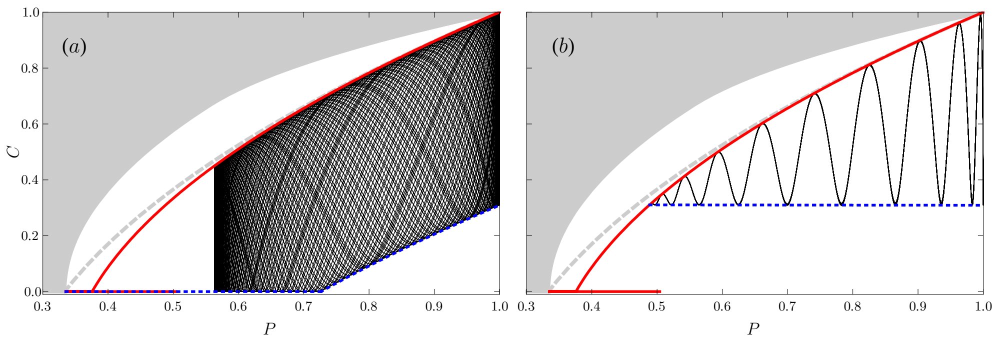

Figure 3. CP-plane, same as figure 2 but for $n = 5$. (a) $\alpha = \pi/20$ for the black and blue line and $\kappa = 5.7$ and $J = 0.2$ for the black line. (b) $\alpha = -\pi/20$ for the black and blue line and $\kappa = J = 5\sqrt{4 \times 5 + 2}$ for the black line.

additional point apart from $C = 1$. This means that there is a small region (hardly visible in figure 3) above the Werner curve that can be reached by the dynamics. We do not write here explicit expressions for the dashed blue curves, as they can be obtained from equations (18) and (19).

Figures 2 and 3 show in black the curves for the interacting case for the same initial states as the blue-dashed curves. One can note that in the CP-plane the curves now form a Lissajous-like figure with their frontier defined by the curves $C_{\pm}^{(n)}(P;\alpha)$ for the starting value of α (lower frontier) and for $\alpha = \pi/4$ (upper frontier). Note that for increasing values of the difference of the interactions the curve in the CP-plane does not fill the entire region enclosed by the curves $C_{\pm}^{(n)}(P;\alpha)$. The region filled by the black curve in figure 2(b) is smaller than in 2(a), because in 2(b) we use larger value of $|\kappa - J|$. The minimum value of the purity can be calculated as $P_{\min} = P(t = \pi/2\omega_n)$; this is the lower bound for P for the black curves in the figures 2(a) and 3(a). A separate case arises when $\frac{1}{2}\arcsin\left(\frac{n^2+n}{3n^2+3n+1}\right) < \alpha < \pi/4$ and

$$(\kappa - J)^2 < \frac{2n(n+1)(3\sin(2\alpha) - 1) + 2\sin(2\alpha)}{2n+1}.$$

The minimum value is $P_{\min} = 1 - 1/\gamma_n$, figure 2($b$). If both interactions have the same strength the curve will fill again the entire area, except in the case when there are commensurable frequencies in F and G, equation (14). That is the case of figure 3(b) where the black curve is closed.

We have given closed solutions for the dynamics of two different TLA in a cavity interacting by dipole–dipole and Ising interactions. Many solvable models discussed for two TLA in a cavity belong to this wider class of exactly solvable models including a model for a Dirac oscillator outside the realm of quantum optics [10]. The effectiveness of the general solution presented was displayed by calculating the evolution of concurrence and purity and fully determining the region of its evolution in a CP diagram in a particular, but interacting case. Interesting features appear when including both types of interactions. Intuitively one might think of less decoherence and a more robust entanglement with increasing interaction between the atoms. This is true if we have either of the interactions, but not necessarily if one takes interactions of similar strength.

The parameter space of the model will be further explored in a full-length paper. Interesting situations include placing one TLA outside the cavity or at a node of the mode and using different detuning as well as coherent or more complicated initial states.

An interesting perspective would be to extend this technique to situations, where the couplings are chosen such that—rather than four-dimensional spaces where exact solutions are available—we would have larger but finite spaces known in molecular physics as polyades, which are accessible to treatments with Lie algebraic [23, 24] and semi-classical [25, 26] techniques.

Another worthwhile line of research may be to find an even more general class of solvable models including the one presented here and the ones using a pseudomode approach [7, 8].

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